

On resolution of double stars

After some years of double star observing it seems time to resume as compact as possible the current state of my "insights" regarding visual resolution of double stars with "small" telescopes. The time spent with the anachronistic passion of visual double star observing was so far a very interesting one as I learned a lot and I am gladly still learning. So it took me for example some time to realise that when looking at a star through a telescope I do not see the star but an optical artefact produced by the telescope with the light coming from the star – the so called diffraction pattern. A fact that we have to keep in mind when trying to resolve double stars.

But first of all we have to define the meaning of "resolution". Calling a double resolved is often understood as clear split with dark space in between the central disks (central part of the Airy disk that can be seen) of primary and secondary. The required minimum aperture for a clear split seems theoretically easy to calculate. While the size of the Airy disk (defined as the radius from peak intensity to first minimum of the diffraction pattern) is independent from the magnitude of a star the size of the visible central disk changes very well with the brightness of the light source. The central disk of very bright stars might occupy 80% or even a bit more of the Airy disk while stars with +5mag occupy already less than 50% and somewhat fainter stars with +9mag about 25% going down to near zero for the really very faint stars. Despite some research I did not find any solid information on the relative size of the central disk depending on magnitude so I did some statistical work myself based on images of open clusters and double stars. Starting with two images of stars with diffraction rings I got a good hint for the relative size of the central disk for these stars with given magnitudes. By counting the pixels to determine the diameter of the central disk I added the data of several other images of open clusters and double stars to get relative sizes of central disks for stars with known magnitudes. I then normalized this data by making the number of pixels for given magnitudes from different images comparable and did then some statistical analysis with the result of a crude brute force algorithm giving the relative size of central disks depending on magnitude. This approach might not have been very precise but to some degree good enough when compared with the results of visual observations so far (download spreadsheet for this calculation <http://www.sterngucker.eu/XLS/Size of spurious disk.xls>).

As the angular size of the Airy disk depends on the diameter of the scope you know now what to look for: The result from adding up both radii from binary and secondary spurious disk should be smaller than the separation of the double in arcseconds to show some dark space between the components in your scope – at least if both components are bright enough to be resolved as single stars and none of the other influencing factors change the rules of the game.

But you need no split to recognize that an object you observe is obviously not a single star – central disks might be "kissing" or slightly overlap, might look like a dented rod, a rod without dents going down to an elongation with heavily overlapping central disks giving the impression of an oval or "egg". To avoid troubles with different concepts of resolution I opt for the obvious recognition of an object being definitely not a single star clear enough to allow a confident estimation of the position of the secondary (without doubts, therefore excluding lucky guesses) as minimum requirement to consider a double as "resolved".

The number of factors influencing double star resolution is as already mentioned large and at least some of them are heavily interacting – in the following I will discuss the in my opinion most important ones.

Physical attributes of the double star:

-- Separation: The smaller the separation the higher obviously the requirements regarding aperture with exponential increase. Larger separations pose less challenge up to none at all with the exception of very faint pairs – secondaries of very wide pairs may be resolved up to near the telescope magnitude limit. When doing some statistical analysis of mostly own observation reports I found the relation X/sep to be a good foundation for an estimation of the for resolution required aperture depending on separation. Treating X as parameter to be determined by the least square method from my data set I noticed that Dawes did a good job – the result was constantly ~ 116 . One special point regarding separation is for close pairs the effect of the position of the secondary in the diffraction pattern of the primary – this will be discussed as separate topic

-- Magnitude of primary: Too bright poses a problem with glare but this is rather rare so I see no reason to deal further with this. On the other side increasing faintness of the primary means an increasing challenge for resolution as is obvious when considering the simple case of a close equal bright double – beginning with magnitudes fainter than +6mag a slow increase of requirements begins up to the degree of non resolution near the telescope magnitude limit. Statistical analysis has also shown a dependence between separation and the difference between telescope magnitude limit and magnitude of primary working as amplifier for this relation – in simple words: How demanding the increasing magnitude of the primary gets depends how close we are to the telescope magnitude with exponential increase depending on separation

-- Magnitude of secondary: This one is obvious – fainter secondaries are harder to resolve up to the degree of non resolution near the telescope magnitude limit. To resolve a companion near the TML a rather large separation is needed and other brighter stars in the field of view can make this even more difficult. Statistical analysis has shown that this effect begins to show with magnitudes fainter than +9mag and it seems to work a bit surprising rather linear

-- Spectral class: In average we expect yellow light with $\sim 550\text{nm}$ wavelength but some stars show a different spectrum. Wavelength has not only an effect on the size of the diffraction pattern but some hues especially reddish ones make stars harder to resolve as they appear visually fainter. The topic gets even more complicated if we consider doubles with different spectral classes for primary and secondary. From a statistical point of view it seems sufficient to stick with the assumption of yellow light and consider variations of colors as "white noise" - but being always aware that a red hue (especially of the companion) makes resolution significantly harder

-- Relations between these factors especially the difference between the magnitudes of primary and secondary (Δ_m): Increasing Δ_m makes it obviously harder to resolve doubles. Statistical analysis has shown that this effect starts with Δ_m larger than 1 with strong exponential impact of separation (in the basic form of Δ_m/sep) and some minor side effects depending on the size of central obstruction: Large central obstruction and large Δ_m seem to be not such a good combination.

Used telescope: Often a hot and usually a bit controversial discussed topic.

-- Aperture: Defines angular resolution limit and magnitude resolution limit of the scope usually indicated in the telescope specifications. Both values are to be taken with caution and are no hard facts but should give an idea what to expect with some spread under reasonable good conditions. Obviously the larger the aperture the better the chances for resolving a given double. As the aperture is usually not a choice but a given it is one of the most important factors for selecting doubles during session planning. Bright doubles with a separation near the angular resolution limit are often considered as most interesting to observe. But also faint and wide pairs near the magnitude resolution limit can offer an interesting challenge

--- Telescope angular resolution limit: Usually given as $116/D_{\text{mm}}$ (Dawes) with D_{mm} for aperture in mm. This is an empirical value derived for small refractors for equal $\sim 6\text{mag}$ bright pairs. If we accept resolution as observed distinctive elongation as discussed above then this value is a bit conservative. Statistical analysis of several successful elongation observation reports has shown that the very lowest resolution limit under else very good conditions might

be around $0.5 \times \text{Rayleigh}$ means $69/D_{\text{mm}}$. On the other side if we demand a clean split then for very bright pairs not even the Rayleigh criterion with $138/D_{\text{mm}}$ is sufficient. How close we get to the angular resolution limit of a scope depends a lot on seeing conditions discussed later on

--- Telescope magnitude limit: Usually given as $2.7+5 \cdot \text{LOG}_{10}[D_{\text{mm}}]$ or similar – it seems obvious that such a formula can only give a very crude hint what might be possible with this scope under very good conditions but all efforts to provide a more precise approach like especially Schaefer's work (see <http://fisherka.csolutionshosting.net/astronote/astromath/SchaeferLMCalc.html>) have only shown how difficult this is. I have now made it custom to start my observing sessions to find the faintest star I can resolve in the target field of view and I found here differences of $\pm 1 \text{ mag}$ depending on seeing conditions. So I know what to expect for my attempts to resolve wide doubles with very faint secondaries – obviously it will then be impossible to resolve fainter than TML secondaries regardless separation. So far I have not found a precise separation value for being able to resolve a secondary near or at TML as this depends very much on given transparency but I think 30" separation is large enough to reduce the challenge of resolving a double to that of resolving a single star. Another interesting impression for close faint pairs: While it is often not possible to get a crisp resolution even with averted view there is often to observe some shimmer like from a nebula – certainly no resolution but a strong hint for being a double

-- Size of central obstruction defining details of the diffraction pattern, especially the size of the Airy disk and peak intensity of central disk and diffraction rings. Also an often hot and controversial discussed topic how good or bad the effects of CO might be for resolving doubles. I did several experiments with different sizes of CO and found the relation between CO and resolution to be not very intuitive. A small CO of ~ 0.2 seems to offer some advantage for resolving close and rather bright doubles with small Δ_m by reducing the size of the central disk, a large CO > 0.35 seems to make resolution of close and very unequal doubles somewhat harder due to the effect of a much brighter diffraction pattern adding an additional challenge to resolve a secondary depending on the position within this pattern. For wide and faint pairs the size of CO seems not relevant and as affordable large apertures often come with a large CO this is the area of double star observing where large reflectors (especially SCTs) are hard to beat

-- General optical quality of the scope usually given either by the producer or by an optical laboratory in terms of Strehl defining peak intensity. Often 0.95 Strehl is considered to be the target line for a scope to be good enough to deliver images without visually noticeable deterioration. Obvious basic line: Good optical quality makes resolution at least for difficult cases easier given everything else equal – but usually this is not such a topic as the quality delivered today is generally very good even for inexpensive scopes

-- Several special topics regarding reflectors (collimation, thermal issues etc.). Basic line: If you want to get a good performance from your scope for resolving difficult doubles then it is absolutely necessary to have no problems in this regard

-- Focal ratio: I am not sure if this is really an influencing factor but higher focal ratio scopes are often praised for crisper resolution – so far I have not much experience in this regard but in the long term I intend to do some comparisons here. Currently I think that there might be no visually noticeable difference – although once I compared directly the image quality of a 60mm mask on my C925 with 2350mm focal length, a 60mm mask on my refractor with 980mm focal length and my 60mm travel refractor with 355mm focal length with the conclusion, that image stability increased strongly with the focal ratio.

Finally the question if the resolution limits of a given scope are a "disadvantage" for observing doubles – I think certainly not because you can use these limitations as kind of instrument for a rough estimation of separation and magnitude. If for example you have checked the current TML of your scope with a specific value in your field of view and you cannot resolve a wide double with a secondary with an advertised brighter magnitude – then you know you have here something to investigate further.

And yes – any scope of reasonable quality is perfectly suited for double star observation within its limits and the number of suitable objects is huge even for very small scopes. But may be there is one exception: Scopes with an noticeable image shift when changing direction of focus (basically all scopes with a movement of the main mirror for focus). When "zooming" in on a double you need to change eyepieces and therefore you need to some degree refocus – image shift might then become a bit irritating.

Atmospheric influences:

This is the source of our permanent failure to ever get even near our theoretical resolution limits.

Best location for visual astronomy would be out in space and second best might be somewhere up in the mountains above the clouds and with rather dry air, but even then there is necessarily an impact caused by the layers of air we are looking through. Atmospheric influences are best checked with the quality of the visual image of a bright star with a magnification high enough to potentially see the diffraction pattern – range goes from a perfect image with crisp central disk and stable diffractions rings to snowballs without any structure and in between are all kinds of fuzzy and jumping images. There exists a lot of extensive theoretical papers and descriptions of experiments on this topic so I keep this short. Basically we have four main factors here:

-- Extinction: The amount of air between observer and space leads to some reduction of the brightness of a star and depending on the altitude of the star. This effect is called extinction and has low impact for the resolution of brighter doubles but some effect on fainter ones near the telescope magnitude resolution limit. This is one good reason to avoid anything below 35° altitude but the most negative effects of low altitude are actually general disturbances of the visual image of the diffraction pattern (called atmospheric dispersion) leading to bad chances for resolving especially of close doubles or even worse to false positives

-- Stability of the air vulgo seeing: The degree of stability of the air is measured according to the stability of the visible diffraction pattern. Several scales exist describing this effect but words alone are not enough to use these scales with some precision but animated images do a good job for example <http://www.damianpeach.com/pickering.htm>. Jumpy and utterly destroyed diffraction patterns are obviously not this good for resolving doubles, especially close ones. But so called bad seeing is often interrupted by fractions of seconds of stability sometimes long and frequent enough to get good results despite "bad seeing" – kind of lucky imaging. So beginning with Pickering ~4-5 we can hope for some good results. But even if we are not "lucky" - bad seeing is no reason to waste an otherwise clear sky: Wide doubles often even under this condition interesting sessions especially near the magnitude resolution limit of the used scope

-- Transparency of the air: High humidity, fog, dust and so on up to some fine sand from the Sahara produce a fuzzy image of the diffraction pattern up to so called snow balls when else visible diffractions rings are joined with the central disk for a fuzzy ball like a globular cluster. Side effects of low transparency are a halo around bright primaries hiding faint secondaries and generally making faint stars still fainter resulting in a reduction of the telescope magnitude limit. A tad of low transparency is sometimes combined with good image stability – a not this bad combination allowing often a good session. But more as a tad of low transparency is really contra productive for resolving close doubles leading to very frustrating sessions if the focus remains on close doubles – wide doubles with companions up to the then reduced TML might save the night. So far I have found no satisfying scale describing transparency but have the impression that the size of halo around a bright star in arcseconds and the relation of the currently observed telescope magnitude limit to the scopes specification are in combination a good measurement of transparency

-- Light pollution: A really dark sky with a naked eye magnitude limit near +6mag is a visual sensation even for non astronomers and a lot of faint deep sky objects need simply a dark

sky to get visible in a scope. Observing double stars has the benefit of low impact of light pollution especially for brighter pairs up to +9mag – even heavy light pollution adds only a few mm to the for resolution required aperture. This changes a lot for fainter stars near the telescope magnitude resolution limit: Together with extinction light pollution might cost up to 10% or even more of the telescope magnitude resolution limit so this means good bye to the really faint fuzzies.

Basic line: Atmospheric influences make the difference between a great double star observing session with spectacular and exceptional results and a modest one with mediocre results and already small changes in conditions may cause a great spread regarding required aperture for resolving a given double up to non resolution at all.

Quality of available double star data: My entry point into double star observing was the Cambridge Double Stars Atlas - really interesting but of limited use for detailed session planning. Then came several online sources like for example the Eagle Creek Observatory with listings of interesting double stars per constellation (<http://www.eaglecreekobservatory.org/eco/doubles/>). Next was the catalog of the probably most prominent double star discoverer – Struve. It took me some time to realise that the data of this catalog was for obvious reasons no longer up to date so finally I landed on the most important data source for double star observers – the Washington Double Star catalog also available online. This time it took me not very long to realise that even here the data has to be taken with care.

The WDS catalog data is strictly based on observation reports – this is clearly a problem for "fast" moving physical pairs. Note code "O" indicates the existence of an entry in the WDS 6th orbit catalog – so this should be checked if the last registered observation is not a recent one.

Then many objects in the WDS catalog are based on systematic search on photographic plates with often crude magnitude estimations – if magnitudes are given without digits then a lot of caution is required, to some degree also if both magnitudes are given with only single digit precision as recent measurements should show double digit precision. And after a while it became clear that this is not only a problem of the WDS catalog but that the simple concept of visual magnitudes is generally somewhat shaky for stars let's say fainter than +10mag – you only have to look up several catalogs for the same star (Hipparco, Tacho II, USNO, UCAC4 ...) to get as many different values as there are catalogs.

Another special topic is the single digit precision of given separations for close doubles as the relative error range increases fast – if 3" is given then "reality" might be between 2.95" and 3.04" if we assume zero error in measurement giving an error range of 3% but the same situation for a 0.3" gives already an error range of 30%.

While the absolute quality of data might seem not this significant when it comes to the quest of resolution it is very well important when it comes to session planning and expectations which aperture might be sufficient. For some time I have now reported obviously wrong WDS data directly to the WDS catalog organisation but in a rather unsystematic way when it just happened to occur.

I have now decided to proceed here in future in a more systematic approach with separate lists of dubious WDS objects per constellation within the reach of my scopes with the goal to eventually weed out WDS errors.

Position of the secondary in the diffraction pattern of the primary: Certainly of relevance for resolving close and faint companions. This is especially true for secondaries sitting more or less centered on the first diffraction ring – can a secondary equal faint as the first ring be resolved as it might for example provide a thickening in this position or will it get lost? If resolution with equal brightness is possible is it then also possible with a somewhat fainter companion? If resolution with equal brightness is not possible to what degree has then the

secondary be brighter than the first ring? Next questions come for the position of the secondary between spurious disk and first ring of the primary and then outside the first ring and so on ...

Diffraction theory does as far as I know not deliver a clear answer in this regard and the existing criterions like Rayleigh, Dawes, Sparrow et al. claim resolution only for equal bright components.

One solid approach to determine the brightness of the first diffraction ring in terms of visual magnitude is certainly to translate the difference in peak intensity into difference in magnitudes according to the logarithmic scale of magnitudes – but it seems questionable to set peak intensity of the spurious disk of a star equal to the peak intensity of a ring at least when it comes to visual observation.

My experience in this regard is so far not very conclusive as there are not this many stars bright enough to show a clear first diffraction ring with a companion close and faint enough to compete with the brightness of the ring. So I am still in waiting position for the next opportunity to directly compare the brightness of a secondary with the brightness of the first ring.

Delta Cyg would for example be a good candidate for such an experiment. I have already resolved once Delta Cyg with an aperture of 70mm thus positioning the companion directly centered on the first ring – but at this time I did not care about this question enough to investigate further. But at least this means that a Δ_m of ~ 3.5 mag is no big problem for resolving a companion centered on the first ring.

If I get this or another similar situation again I will then apply some central obstruction to the scope thus increasing the brightness of the first ring. And as diffraction theory is nice enough to keep the radius of the first ring approximately unchanged despite the change in the size of the Airy disk this should then allow to get a clear relationship between the brightness of the first ring and the brightness of the secondary at the point of the secondary being no longer resolvable with a specific CO size.

If interested in such questions: Download here

(<http://www.sterngucker.eu/XLS/Position%20of%20Secondary%20in%20the%20Diffraction%20Pattern%20of%20the%20Primary.xls>) a spreadsheet giving the required apertures in mm to have the secondary at specific positions in the diffraction pattern of the primary. The given values are valid for refractors – but as I said the radius of the first ring remains nearly unchanged at least for visual observation. So any close double with a primary bright enough to provide a visible first ring with a Δ_m of 3 or better more should provide an interesting target for this topic.

Personal attributes of the observer: Several factors might be relevant if a given double might be resolved or not:

- Experience: The first look through a telescope is often (especially when unguided) rather disappointing – there are all these images in the journals and the internet with incredible colors and details and then all you see is a grey blotch. Then the learning curve starts getting slowly a bit steeper, so some experience is necessary to be able to see details invisible to the beginner. And over time some more or less individual techniques evolve for handling difficult cases: Use of averted vision, moving the target purposely through the field of view, changing between intra-focal and extra-focal, changing magnification back and forth (with not this good seeing less is often more) ...

- Human optical system: The effects of eyes and brain on resolution. Lots of scientific papers are available on this topic but this is certainly not my field, so I keep this extra short:

-- Age: Exit pupil plays an important role and according to Schaefer's work on magnitude limits age is actually an advantage. So any complaints like "My old eyes ..." are to be considered as fishing for compliments

- Personal acuity: Usually defined by the magnification needed to split a given equal bright double with given separation – for example to split a 2" equal double with x80 gives a personal acuity of 160"
- Ability to detect minimal different shades of grey in terms of spots against the background – vulgo limiting magnitude of the eye
- Ability to detect faint spots near brighter ones: This is may be the other side of the same coin – less sensibility to glare might be an advantage here.

Other factors: So far we have covered most of the resolution relevant factors but this list can never be complete it seems.

At least there is a lot of less common specific conditions: Primary with excessive glare, multiples with a faint component between two brighter ones, specific combinations of colors for example blue white for the primary and reddish for the secondary, doubles near another bright star etc.

Basic line: It is impossible to consider all factors relevant for resolution so any double might pose a specific challenge.

Session planning:

Looking through a telescope without having a plan in advance what to observe means asking for frustration. You might rely on your electronic equipment proposing some "best of the night" targets but just looking at objects however beautiful they might be without knowing some facts in advance gets boring at least in the long term. As a starter for double star session planning you might use the many sources available online:

Beginning with simple lists of "highlights" like "Top 200 most beautiful Double Stars" (<http://users.compaqnet.be/doublestars/>) or the list of "Best Multiple Stars" from the SAGUARO ASTRONOMY CLUB (<http://www.saguaroastro.org/content/BEST-MULTIPLE-STARS.htm>) to more comprehensive lists like the "Double Star Listings" from the Eagle Creek Observatory (<http://www.eaglecreekobservatory.org/eco/doubles/>).

If you are interested in colorful doubles then "85 multiples of unusual color" (<http://www.raycash.org/85mult.htm>) might be a good starting point.

If you are interested in the very details of the doubles you might observe then "Star Splitters" (<http://bestdoubles.wordpress.com/>) is a website highly to recommend.

Another possibility is to join projects started by others like "The Spirit of 33" (<http://www.carbonar.es/s33/33.html>) with the target to observe 33 doubles per constellation or the "Double Star Program Observing List" of the the Astronomical League Double Star Club (<http://www.astroleague.org/al/obsclubs/dblstar/dblstar2.html>) or the "The Year-Long 60mm Telescope Challenge" (http://www.cloudynights.com/item.php?item_id=2862).

If you are impressed with the work of famous double star discoverers you might work through the complete lists of for example Struve with 4134 objects or Burnham with 1540 objects and compare your observations with the current data in the Washington Double Star catalog (<http://www.usno.navy.mil/USNO/astrometry/optical-IR-prod/wds/WDS>).

If you are interested in questionable objects in the WDS catalog you have the separate catalogs "WDS Neglected Doubles" available (<http://ad.usno.navy.mil/wds/wdstext.html#neglected>) as base for your planning.

You might even try to discover new double stars – this is not this impossible as you might think: Just systematically search the UCAC4 catalog within the magnitude limit range of your scope for close objects and check then this positions for an entry in the WDS catalog – if no WDS object exists for this position you have a valid starting point for further research and hopefully a successful first visual observation of a "new" double star.

Sketching and imaging (see for example "Double Star Imaging Project" at https://groups.yahoo.com/neo/groups/double_star_imaging/info) and measuring double stars might offer an interesting challenge in addition to visual observation.

These are only a few possibilities and there are many more valuable resources available in form of books (Cambridge Double Star Atlas or Haas' book on Double Stars for Small Telescopes) or in a vast number of websites.

Base line: Detailed session planning is highly recommended and the WDS catalog is the most versatile data source to do this. A side benefit of detailed session planning is constant learning about your sky and the next best thing to a good observing session is planning the next sessions during foul weather. And even if the weather does not cooperate and time goes by without a chance to execute a session plan – stars have the nice property of coming again next season and without a few exceptions (fast orbits and new measurements) double star session plans are valid for years to come.

As seeing conditions (especially seeing and transparency) are seldom known in advance it seems wise to have alternative session plans for different conditions available or at least include in standard plans also objects of interest suitable for not this good seeing conditions. To some (but really only to some) degree it is like with weather: There is no such thing as bad seeing conditions, there are only wrong session plans. Against cloudy nights there is no remedy, but even then fast moving clouds may offer spectacular and dramatic sights of the moon with a small scope or binoculars.

How to do session planning is as indicated above certainly highly individual depending on available equipment and tools and above all special interests and agendas. In my opinion some agenda is needed to keep long term interest alive but that's just a personal attitude.

My own current personal agenda regarding double star observation is threefold:

- Observing as many doubles as possible coming over the year into my field of view for my given location and the range of my equipment (from a 60mm refractor travel scope up to a 235mm SCT with several scopes in between)
- Determine the needed aperture for resolving these doubles under average conditions
- Weeding out as many obvious errors in the WDS catalog as possible (again within the range of my equipment).

As an example how session planning might be done I describe my procedure without claim that this is the best and only way especially when considering my preferred setup with old fashioned Alt/Az mount without any electronic equipment means finding targets strictly visually by star hopping:

- Selecting doubles from the WDS catalog per constellation with the criterions separation larger than 0.5" (half radius of Airy disk from my largest scope) up to a maximum of 30" (larger separations are hardly to realise visually as doubles) and magnitude for the secondary up to +12.5mag (as I have to consider some light pollution). Depending on the size of the constellation this usually results in several hundred potential targets. I do this offline using session planning software with a rather recent (currently 2014.5) local version of the WDS catalog. If you don't have such a tool available you might download a spreadsheet containing the WDS catalog for example from Bruce's website (<http://www.handprint.com/ASTRO/XLSX/WDS2014.xlsx>). This contains no constellations so you might have to organise your sessions plans by RA/Dec areas. Another possibility is using an online search machine like for example Stelle Doppie (http://stelledoppie.goaction.it/index2.php?section=2&azione=ricerca_avanzata). As the WDS catalog is heavily maintained (the WDS entries are based on observations and there seems to be a constant feed as the data for a reasonable percentage of the WDS objects is changed each year) it is to recommend to check questionable cases directly online with the WDS website
- For WDS objects with a note code "O" I check the WDS Sixth Catalog of Orbits (<http://ad.usno.navy.mil/wds/orb6/orb6ephem.html>) especially if the last recorded observation is not of the current year
- Additionally I check for Struve and Burnham objects in the constellation in question to be sure not to miss any double from these discoverers

- Then I select stars in the constellation bright enough (most perfect is anything brighter than +2.5mag, under very good conditions this might go up to +3.5mag) to be found with the naked eye even under severe light pollution to be used as starting points for sessions. Rather often such stars are also very wide doubles, so this is then an exception for my limit of 30" separation
- Beginning with these entry points I start then to group the selected objects by their coordinates (using software giving a field of view populated only with the selected WDS objects) to be located by star hopping within field of view segments of 3° (about the largest field of view I can get with my 120mm refractor). Usually there are several plans possible starting with the same entry point to deal better with a large number of available objects. In case of missing objects in a specific field of view I look again at the WDS catalog for objects outside my standard selection criterions to be used as hops and only if there are no doubles available I select brighter single stars as hops
- I try to keep the number of objects in a plan less than 20 as the resolving of doubles needs some concentration and my personal limits for this task seems to be about 15 objects. Often this is not possible as there are simply too many targets fulfilling my selection criteria – I try then to split a large session plan into two smaller ones else I will end up with session plans never to be completed. A work around is to look for shortcuts to speed up the process when I get the chance for a second attempt
- For WDS objects with a single digit magnitude for the companion I assume crude estimation instead of precise measurement and check with the UCAC4 catalog. I use here the same session planning software with a local UCAC4 catalog – if the separation is larger than 2" then in most cases there exists an UCAC4 object for the secondary allowing a countercheck for the magnitude (there is no direct measurement of visual magnitude available in UCAC4 but only a calculated value based on the measurements in several other bands – but UCAC4 seems to be the most reliable source when it comes to magnitudes fainter than +11.5mag. If you don't have a local UCAC4 catalog available you might check the Uni Heidelberg website searching UCAC4 objects by position (<http://dc.zah.uni-heidelberg.de/ucac4/q/s/form>). Very useful is also Tom's web service (http://mainsequence.org/html/wds/shData4_WDS_Pair/shWDS_Pair.html) to compare WDS and UCAC4 data by discoverer ID. In cases of single digit magnitude for doubles with <2" separation it makes also sense to check the combined magnitude usually available as Tycho II object in this position (download spreadsheet to do this: <http://www.sterngucker.eu/XLS/Combined%20Magnitude.xls>) – if the calculated value is brighter than the Tycho II magnitude then wrong data might be suspected
- The mentioned bright entry point into a session plan has three additional functions: First to determine seeing quality according to Pickering by increasing magnification to get an impression how stable the diffraction pattern is, second to determine transparency by estimating the size of the halo around the bright star, third to determine the telescope magnitude limit in the current field of view on base of a 20-30' FoV star map with all faint stars up to +13.5mag based on the UCAC4 catalog
- After selecting the objects suitable for a session plan I make then star maps using Stellarium for the Alt-Az view from my location with horizontal flip as I use a diagonal and an image processing program allowing making the image inverted and monochrome. These star maps I use then during my sessions
- Finally a list of the objects is created with the basic data of the objects including additional informations if the WDS data seems questionable when compared with other star catalogs. A specific information is the for resolution proposed aperture just for field testing of my current Rule of Thumb approach. There is clearly also room for notes during the sessions or even a quick sketch if the situation requires it.

When executing session plans I note the results per object (resolution or not and if resolved: Position at a fictive clock, needed magnification for resolution with the used telescope, minimum aperture needed for resolution if the used scope is equipped with an iris diaphragm

or aperture masks). After finishing the session I check per double if the noted position is correct with an allowance of $\sim 15^\circ$ as criterion if the resolution was correct and add in this case the noted minimum aperture to my data set of limit observations. If the noted minimum aperture does not match the expectations at all or if no resolution was possible although using an aperture far larger than proposed additional research begins if the current WDS data might require correction. Used tools for doing such research are for example "Aladin Sky Atlas" (most comprehensive combination of images including 2MASS and catalogs I know so far – available directly in your webbrowser with <http://aladin.u-strasbg.fr/AladinLite/examples/full-screen.html> or as local program on your desktop or notebook with download from <http://aladin.u-strasbg.fr/java/nph-aladin.pl?frame=downloading>) or the Sky-Map.org website (<http://www.sky-map.org/>) combining different image sources like DSS and SDSS-III with mouse over function for the Tycho II and USNO catalog. Other resources are SIMBAD and naturally the Cloudy Nights Double Star Observing forum.

This research might settle open questions or finally end with an according communication with the WDS organisation to get things as straight as possible.

Rule of Thumb (RoT) for an proposed aperture for resolving doubles:

Many attempts have been made to get a grasp on calculating the needed aperture for resolving a given double star (or the other way around the minimum separation resolvable with a given aperture) starting with criterions for equal bright stars like from Dawes or Rayleigh. While Rayleigh's suggestion is based on optical theory (usually given for yellow light to eliminate the question of spectral class) the Dawes' criterion is based on an average value derived from a set of observations. The Dawes criterion "resolution limit in terms of separation in arcseconds = $116 / \text{Aperture in mm}$ " ($s=116/D_{\text{mm}}$) is even used as resolution limit in the technical specifications of telescopes giving this value an impression of precision. But it is obvious that there has to be for statistical reasons some spread around this value derived as average from a dataset. I don't know here any indication in this regard from Dawes himself but personal experience suggests a standard deviation of $\sim 14\%$ around the mean value 116. This is also supported by an empirical found lower limit for resolving doubles at about half the Rayleigh criterion as observation reports have shown for rather bright pairs under excellent conditions distinctive elongations allowing estimation of position and separation with confidence down to this value. This corresponds very well with the above mentioned standard deviation value and this means that if we sample for example observation reports for equal bright doubles up to +6mag with a separation of 1 arcsecond under reasonable fair conditions 2/3 of the apertures of all positive reports would be in the range 100–132mm, 95% in the range 84–148mm and 99,5% in the range 68–164mm.

So far for more or less equal bright doubles up to +6mag. Even if we know that also magnitude significantly different from +6mag plays a significant role for resolution we might take this as base for ideas regarding resolution of unequal doubles. The instant idea popping up here is that the difference in magnitude between primary and secondary = Δ_m might be the most important factor besides separation. This suggests a simple modification of the Dawes criterion in the form of $s=116/D_{\text{mm}}*\Delta_m$ as proposed by Bruce and Fred on the Cloudy Nights Double Star Observing forum – this would mean a linear increase of separation and therefore aperture by Δ_m . For equal bright doubles (with Δ_m less 1) we fall back to Dawes by setting Δ_m to 1. To determine the proposed aperture for resolving a given unequal double we can change this formula to $pD_{\text{mm}}=116/s*\Delta_m$. A few tests with selected unequal doubles show quickly that this simple approach might work well for bright and not too unequal doubles but is overall a rather poor performer as it ignores the magnitudes of the components of the double – and proposing the same aperture for resolving a +5/7mag pair as well as a +9/11mag pair is obviously nonsense. Besides the assumed linear relationship between separation and Δ_m is obviously an oversimplification.

This means we have to look for more advanced models – at least three come to my mind: The "Fuzzy Difficulty Index" (<http://www.carbonar.es/s33/Fuzzy-splitting/fuzzy-splitting.html>) and the attempts from Chris Lord (<http://www.brayebrookobservatory.org/BrayObsWebSite/BOOKS/TELESCOPIC%20RESOLUTION.pdf>) and Napier-Munn (www.jdso.org/volume4/number4/Napier_Munn.pdf).

All these models have several shortcomings for different reasons:

- The "fuzzy" approach basically ignores the question of aperture and is thus of no use to determine the for resolution required aperture
- Lord's model structure is also not of this good use when asking for a proposed aperture for a given double as he uses aperture classes making this question a recursive one. But the main weakness of his model is the fact that he also ignores the effect of fainter magnitudes similar to the above described simple RoT approach. He even argues that this has little effect even if the contrary is obvious. Else his work is very interesting and shows a deep knowledge of diffraction theory
- The Napier-Munn model is based on statistical analysis of several hundred observations and the concept of working with probabilities for resolution with a given aperture seems very interesting to me especially in the form of asking for the aperture with a 50% probability for resolution. But the algorithm based on his model structure has an obvious numerical problem, delivering for many test pairs an error as the proposed aperture for a 50% resolution probability would be less than zero what is obviously nonsense
- Lord and Napier-Munn work both with a data set of observations but all of them are to my knowledge with fixed apertures – so only by chance might several observations really on the limit regarding aperture but most of them are not. So in my opinion the used data sets are in both cases not up to the intended task and especially the data set used by Lord seems outdated and thus of no good use for serious statistical analysis.

What despite these shortcomings these models show nicely for me is that there is no such thing as a limit in terms of required aperture as one precise number but an aperture range with a probability distribution – for reasons of simplicity assumed symmetrical although it is evident that there is more room in needed aperture up than down. In reality there is no such thing as an upper limit as conditions may be this bad, that no amount of aperture may be of help - problems with seeing might even work the other way around giving an advantage to the smaller aperture.

Observation reports for selected pairs from different observers (see the Sissy Haas project: http://www.billboubnitz.com/Haas_Project/hbsop_index.html) show there is with the exception of the very easy doubles usually a wide range of overlapping apertures with positive and negative reports reflecting different circumstances when observing. And my own observation logs show such a range also for only one and same observer mostly depending on differences in seeing conditions.

As the number of influencing factors seems vast any attempt for a complete analytic model seems futile (as Schaefer's attempt for a complete analytical model for the telescope magnitude resolution limit has shown) - so in my opinion a statistical approach combining some basic theoretical optical concepts with numerical curve adapting might be the best approach.

Basic question remains, which factors to include in such a model - this depends certainly on the intended use of such a model and the informations one can usually expect from observation reports.

For session planning the in advance known parameters are

- the data for the double

- o separation
- o magnitudes of both components
- o delta_m
- the data of the scope available
 - o aperture
 - o size of CO
- average light pollution in the given location
- average extinction for selected field of view.

Next step is then the sampling of as many observation reports of double star resolutions with smallest possible aperture for the given conditions. This might be some with fixed apertures but obviously at the "limit" but mostly such done with variable aperture (with the help of aperture masks or iris diaphragms) to be for sure on the "limit". And obviously we are not asking for the one and only "true limit" observation done under perfect conditions but for the many different results reflecting the very stochastic behavior of photons. Meanwhile I have a data set of several hundred observations of this kind available but only up to 140mm aperture and I intend to proceed with this approach with a new scope up to 200mm. This might then be large enough to cover the usual amateur range for double observing. Larger apertures would be at reasonable costs only available as reflectors making the use of aperture masks difficult as then the central obstruction gets quickly too large to allow reasonable double star observing.

Next step is then deriving a model based on some knowledge in optical theory and calculating the parameters of this model with the help of nonlinear regression analysis, looking at the results and adapting the model step by step to a result with a reasonable small standard deviation and a correlation coefficient near 1 as quality parameters. With an earlier (means smaller) version of the current data set I have done this with the following result:

pD_mm = proposed aperture diameter in mm with a resolution probability of 50% (telescope Strehl 0.95 or better, reasonable good seeing, reasonable good transparency, average personal acuity assumed) and a standard deviation of 14%.

The structure for the model is an addition of submodels in the form of $\text{base} + f(\text{delta_m}) + f(M1) + f(M2) + f(\text{NEML})$ followed by a TML-check with submodels as follows:

Base = Dawes criterion $116/s$ as base modified depending on size of central obstruction reducing size of Airy disk

$f(\text{delta_m})$ = function of delta magnitudes in relation to separation including negative influence of increasing CO. Current version of $f(\text{delta_m})$: $\text{pr1} * \text{delta_m} / \text{sep}^{\text{pr2}} * (1 + \text{CO})^{\text{pr3}}$ if $\text{delta_m} > 1$ else zero with pr1, pr2 and pr3 being parameters to be determined by nonlinear regression analysis

$f(M1)$ = function of the magnitude of the primary depending on separation. Current version $f(M1)$: $\text{pr4} * (M1 + \text{pr5}) / (\text{sep}^{\text{pr6}})$ if $M1 > 6$ assuming that this function has to work only for primaries fainter than +6mag else zero. To makes things a bit complicated there is the need to counterbalance the exponential effect of smaller separations – so there is an decrease of this function necessary if the separation is smaller than the difference between the constant 14 minus M1. I have no good explanation for this value 14 but it is well supported by the existing data - and pr4, pr5 and pr6 are again parameters to be determined by nonlinear regression analysis. Just another switch is necessary to stop this subfunction at $M1 = +12.5\text{mag}$ as I see no need to go below this value

$f(M2)$ = function of the magnitude of the secondary: $\text{pr7} * (M2 - 9)$ if $M2 > 9$ assuming that this function has to work only for secondaries fainter than +9mag. This assumption is backed by

the existing data and pr_7 is again a parameter to be determined by nonlinear regression analysis

$f(\text{NEML})$ = function of Naked Eye Magnitude Limit for a given location including extinction in the field of view (and not zenith as usually used): $pr_8 * (6.5 - \text{NEML})$ with the assumption that this has to work only for doubles with secondaries fainter than +9mag and pr_8 is again a parameter to be determined by nonlinear regression analysis. The constant 6.5 assumes that the perfect sky offers a NEML of +6.5mag.

TML-check: After calculating pD_{mm} comes a check if the proposed aperture diameter is large enough to resolve the secondary as single star depending on TML – if not an accordingly larger value is calculated giving then pD_{mm}' = proposed aperture diameter after check against the for light pollution and extinction adapted telescope magnitude limit. This is done for all doubles in an approximation process but is usually only relevant for doubles with companions fainter than +10mag. This is then the final value for the proposed aperture diameter pD_{mm}' if larger than pD_{mm} of the first step. The current implementation is very optimistic it seems so I work here mostly with NEML 2.5 to get a bit more realistic results but even then these are often still rather too optimistic. This is probably also due to the fact that I have to calculate from the given aperture for the given NEML – but reality shows that there is a wide band of TML variation of up to 1.2mag under seemingly ident conditions.

Required input for the RoT calculation therefore: Size central obstruction for scope planned to use, naked eye magnitude limit for specific location and average extinction for given altitude, double star separation, magnitude of primary, magnitude of secondary.

Intentional limitations: Primary up to +12.5mag, secondary up to 14mag and size of CO up to 0.4 – these values are considered to be on the upper limit to be of use for amateur astronomers.

Other known limitations: Several specific conditions like multiples with close faint companions in between brighter ones or very bright primaries with glare, humidity in the air giving halos around brighter primaries or doubles against bright nebulas, secondaries in the red color spectrum etc. are not covered by the algorithm but only by the probability concept - means bad luck.

Known weaknesses: The final TML check is currently too optimistic meaning pD_{mm}' is often calculated too small for a realistic 50% chance for resolution of very faint companions. Also the influence of CO is currently implemented not precise enough. Experiments have shown, that a small amount of CO of ~0.175 might be a peak value for positive effects and ~0.25 might be the in average the value where the negative effects begin slowly to evolve with a serious visual impact beginning with 0.35.

Planned enhancements: Enlarge the data set of limit observations for better statistical significance especially for apertures larger than 140mm, smoother transition of the different components of the model when crossing the defined thresholds (especially 6 for M1 and 9 for M2). Better implementation of the effects of CO values. Better implementation of the TML check.

The current beta version of the RoT model can be downloaded as spreadsheet from <http://www.sterngucker.eu/XLS/WRAKs%20RoT%20V3.3%20Beta.xls>.

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